1878MNRAS..38..457N

would nowadays be considered worthless. The probable error of their contacts, instead of being 3^s, as Mr. Stone thought nine or ten years ago, was more likely 20^s or 30^s. I consider that the observations of the Transit of 1769 are fairly satisfied, taking into account the instruments used and the errors actually committed in 1874, by any parallax between $8''{\cdot}60$ and $8''{\cdot}95$. The equations given by Mr. Stone (*Monthly Notices*, vol. xxviii., p. 263) enable anyone with great facility to find the effect of "doctoring" the times of duration.

On some Terms of Long Period in the Mean Motion of Mars.

By E. Neison.

I have lately discovered two new terms of long period in the mean motion of the planet *Mars* which have coefficients of sensible value, and whose existence will in part explain the present errors in Leverrier's Tables of *Mars.* These terms are due to the second power of the disturbing forces, and arise in the main from the action of one planet on the perturbations produced by another. Their value has been calculated by a very similar method to that employed by Leverrier in computing the value of the two terms of similar nature which he has included in his Tables of *Mars.* In general, however, it has been found advisable to use the perturbations of the co-ordinates rather than the perturbations of the elements. For, as a rule, if the former be employed the true value of the resulting coefficient will be given at once, whereas if the latter be used, the value will result from the addition of two large terms with contrary signs, each resulting from a distinct calculation. The values of the coefficients of the disturbing function, and of the perturbations of the elements or co-ordinates, have been derived from Leverrier's Memoirs in the *Annales de l'Observatoire de Paris.* Leverrier's notation is employed throughout.

The first of these terms is due to the combined action of the Earth and *Saturn* on the motion of *Mars.* It has the considerable period of $220{\cdot}90$ years, and the value of

$$\begin{array}{llll}
-0''{\cdot}3388 \sin\,([2n'''-n''-2n^{\mathrm{v}}]\,t & +\,2\epsilon'''-\epsilon''-2\epsilon^{\mathrm{v}} & +\,\varpi''') \\
+0{\cdot}0250 \sin\,(\qquad\quad ,, & \qquad\quad ,, & +\,\varpi^{\mathrm{v}}) \\
+0{\cdot}0248 \sin\,(\qquad\quad ,, & \qquad\quad ,, & +\,\varpi'')
\end{array}$$

Adding and reducing to numbers and it becomes

$$+0''{\cdot}3673 \sin\,(1^\circ{\cdot}630\,[\mathrm{Y}-1852{\cdot}08]).$$

The value of this term in the mean motion of the Earth is given in a short paper of mine in the *Monthly Notices* for

1878MNRAS..38..457N

November 1877. In that paper, my object being merely the perturbations in the motion of the Earth, and not in that of *Mars*, I contented myself by merely reversing the value for the principal term in the motion of the Earth, and thus obtained only a fraction of the complete value in the motion of *Mars*. The existence of this new term will appreciably affect the mean motion of *Mars* as determined by Leverrier, possibly by 0″·01, and in this case it would, at the present time, alter the position of *Mars* by 0″·25, in addition to the more direct action which the term itself will have.

The second term is still more important, for its effects cannot be eliminated by a change in the mean motion. It arises from the combined attraction of the Earth and *Jupiter* on the motion of *Mars*, and it has the comparatively short period of 46·52 years. It has the value of

$$\begin{aligned}&-\overset{''}{0}{\cdot}2934 \sin ([6n''' - 3n'' - 2n^{\mathrm{iv}}]\, t + 6\epsilon''' - 3\epsilon'' - 2\epsilon^{\mathrm{iv}} - \varpi'''),\\ &+0{\cdot}0530 \sin (\qquad ,, \qquad\qquad ,, \qquad - \varpi''),\\ &+0{\cdot}0025 \sin (\qquad ,, \qquad\qquad ,, \qquad + \varpi^{\mathrm{iv}} - 2\varpi''').\end{aligned}$$

Uniting these separate portions, and reducing to numbers, the term becomes

$$\overset{''}{0}{\cdot}3285 \sin (\overset{\circ}{7}{\cdot}740\, [Y - 1835{\cdot}75]).$$

This term will therefore change the place of *Mars* by two-thirds of a second in the short period of twenty years, so that in this term alone the place of *Mars* will be affected by this amount since the epoch of Leverrier's Tables. Further, it is probable it will still further alter Leverrier's value of the mean motion, for although its short period would prevent this in a long series of observations, where much weight is attached to the modern observations it will have some influence.

In considering the influence of these terms in the mean motion of *Mars*, it must not be forgotten that the values which have been given are in heliocentric co-ordinates. The first term can alter the geocentric longitude of *Mars* by nearly two seconds of arc, and the second term by more than one and a half second, and as these will systematically affect the observation for several years at a time, they will become very sensible quantities. Thus, from the combined effect of these two terms, the geocentric place of *Mars* during opposition thirty years hence will be four seconds greater than it was at the commencement of the century.

During my investigation I have come across a number of other terms of long period in the mean motion of *Mars* with coefficients of less magnitude than those which have been already mentioned, but with values sufficiently great to render them sensible in delicate observations of *Mars*. They are, in fact, more important than a great number of the terms which are included

1878MNRAS..38..457N

by Leverrier in his Tables, and as they might all be included in a single table of single entry, it appears to me that they might be included with advantage in the Tables of *Mars*.

These terms are the following, the epoch being 1800:—

$$+0''{\cdot}0458 \sin\{[2n' - n - n^{\text{iv}}]\,t + 283^\circ\,24\} \quad \text{or} \quad +0''{\cdot}0458 \sin\{7^\circ{\cdot}53\,[\text{Y} - 1836{\cdot}60]\},$$

period 46·75 years. This term will augment the preceding term, and be confounded with it for a long period.

$$+0''{\cdot}0420 \sin\{[4n''' - 2n'' - n^{\text{iv}}]\,t + 158^\circ\,36'\} \quad \text{or} \quad +0''{\cdot}0420 \sin\{15^\circ{\cdot}28\,[\text{Y} - 1860{\cdot}33]\},$$

period 23·37 years.

$$-0''{\cdot}1024 \sin\{[4n''' - 2n'' - 2n^{\text{iv}}]\,t + 206^\circ\,48'\} \quad \text{or} \quad +0''{\cdot}1024 \sin\{15^\circ{\cdot}07\,[\text{Y} - 1861{\cdot}48]\},$$

period 23·88 years. This term, with that preceding, will practically form a single term for many years, and this compound term will have a coefficient of 0''·15, and be capable of systematically altering the geocentric place of *Mars* by three-quarters of a second of arc in the very short period of 10 years. All these three, as well as the next term, arise from the combined action of the Earth and *Jupiter*.

$$+0''{\cdot}0230 \sin\{[13n''' - 7n'' + n^{\text{iv}}]\,t + 334^\circ\} \quad \text{or} \quad -0''{\cdot}0230 \sin\{1^\circ{\cdot}368\,[\text{Y} - 1781]\},$$

period 262·2 years. Despite its long period this term is very small.

The next three terms arise from the combined action of *Jupiter* and *Saturn* on the planet *Mars*.

$$+0''{\cdot}0243 \sin\{[n''' - 7n^{\text{iv}} + 2n^{\text{v}}]\,t + 314^\circ\,43'\} \quad \text{or} \quad +0''{\cdot}0243 \sin\{3^\circ{\cdot}123\,[\text{Y} - 1814{\cdot}50]\},$$

period 115·30 years.

$$+0''{\cdot}0536 \sin\{[n''' - 8n^{\text{iv}} + 4n^{\text{v}}]\,t + 199^\circ\,30'\} \quad \text{or} \quad +0''{\cdot}0526 \sin\{2^\circ{\cdot}505\,[\text{Y} - 1807{\cdot}79]\},$$

period 143·70 years. Both these terms have small coefficients from the different portions neutralising each other.

$$+0''{\cdot}1145 \sin\{[n''' - 10n^{\text{iv}} + 9n^{\text{v}}]\,t + 193^\circ\,52'\} \quad \text{or} \quad +0''{\cdot}1145 \sin\{2^\circ{\cdot}100\,[\text{Y} - 1806{\cdot}62]\},$$

period 171·60 years. This term, with that preceding, will unite for many years to form a compound term with a coefficient of 1''·7, and capable of sensibly altering the position of *Mars*. They will slightly affect the value of the mean motion determined by Leverrier.

The last term is due to the action of *Venus* and the Earth on *Mars*.

$$-0''{\cdot}0900 \sin\{[8n''' - n' - 2n'']\,t + 52^\circ\,1'\} \quad \text{or} \quad -0'{\cdot}0900 \sin\{0^\circ{\cdot}874\,[\text{Y} - 1740{\cdot}52]\}.$$

It has a period of 411·7 years.

1878MNRAS..38..460A

The terms depending on the argument $[n' - 4n''' + 6n^{\text{IV}}]$ and $[n''' - 9n^{\text{IV}} + 7n^{\text{V}}]$ are insensible. The term arising from the combined action of the Earth and *Saturn* with the argument $[19n''' - 10n'' - 3n^{\text{IV}}]$, or of the 6th order of the eccentricities, has the enormous period of 6,000 years, and it has a coefficient of over a second of arc. Its enormous period renders this insensible. It is remarkable, however, for it will be a secular term when the motions of the perihelion are considered. Apart from other considerations, it has been thought well to place the values of these terms on record for the purpose of saving useless labour to subsequent investigators.

Note on a Remarkable Property of the Analytical Expression for the Constant Term in the Reciprocal of the Moon's Radius Vector. By Professor J. C. Adams, M.A., F.R.S.

Let $nt + \varepsilon$ denote the mean longitude of the Moon at the time t; $n't + \varepsilon'$ that of the Sun.

$\xi = nt + \varepsilon - n't - \varepsilon'$, the mean elongation of the Moon from the Sun.

ϕ, the Moon's mean anomaly.

ϕ', that of the Sun.

η, the Moon's mean distance from the ascending node.

$c = \dfrac{d\phi}{ndt}$ and $g = \dfrac{d\eta}{ndt}$, so that $(1 - c)\,n$ denotes the mean motion of the Moon's perigee, and $(g - 1)\,n$ denotes the mean retrograde motion of the Moon's node, in a unit of time.

Also let e denote the mean eccentricity of the Moon's orbit.

e', the eccentricity of the Sun's orbit.

γ, the sine of half the mean inclination of the Moon's orbit to the ecliptic.

$m = \dfrac{n'}{n}$, the ratio of the mean motion of the Sun to that of the Moon.

μ, the sum of the masses of the Earth and Moon.

$a = \left(\dfrac{\mu}{n^2}\right)^{\frac{1}{3}}$, the mean distance in the purely elliptic orbit which the Moon if undisturbed would describe about the Earth in its actual periodic time.

To fix the ideas, we will suppose the quantities e and γ to be defined as in Delaunay's Theory of the Moon.

If r denote the Moon's radius vector, and if we omit terms depending on the Sun's parallax, then, as is well known, the value of $\dfrac{a}{r}$ may be expanded in an infinite series involving cosines of angles of the form

$$2i\xi \pm j\phi \pm j'\phi' \pm 2k\eta,$$